

## PAPER

# QoS-Guaranteed Wavelength Allocation for WDM Networks with Limited-Range Wavelength Conversion

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**SUMMARY** In this paper, we consider QoS-guaranteed wavelength allocation for WDM networks with limited-range wavelength conversion. In the wavelength allocation, the pre-determined number of wavelengths are allocated to each QoS class depending on the required loss probability. Moreover, we consider two wavelength selection rules and three combinations of the rules. We analyze the connection loss probability of each QoS class for a single link using continuous-time Markov chain. We also investigate the connection loss probability for a uni-directional ring network by simulation. In numerical examples, we compare connection loss probabilities for three combinations of selection rules and show how each combination of selection rules affects the connection loss probability of each QoS class. Furthermore, we show how wavelength conversion capability affects the connection loss probability. It is shown that the proposed allocation with appropriate wavelength selection rule is effective for QoS provisioning when the number of wavelengths is large. We also show the effective combination of wavelength selection rules for the case with small wavelength conversion capability.

**key words:** *QoS provisioning, wavelength allocation, wavelength routing network, limited-range wavelength conversion, continuous-time Markov chain*

## 1. Introduction

In all-optical wavelength routing networks, connections are established by wavelengths between end nodes and data is transmitted with the connections [1]–[6]. Without optoelectronic-optic (O/E/O) conversion, a connection is established along several intermediate nodes which consist of optical switches with a capability of wavelength routing [7]. If nodes do not have the capability of wavelength conversion, the same wavelength is required at each link to establish connection between end nodes (*wavelength continuity constraint*) and the resulting connection blocking probability increases.

In [8], connection blocking probability in a wavelength routing network without wavelength conversion has been considered with an M/M/c/c queueing model. [7] has investigated the blocking probabilities of distributed wavelength assignment (DWA) algorithms in which random assignment algorithm and locally-most-used (LMU) algorithm have been considered with M/M/c/c based blocking models for ring networks.

On the other hand, if nodes have the capability of wavelength conversion, connection blocking probability is im-

proved. In [9]–[11], the impact of wavelength conversion has been studied with analytical models and simulation.

Under the current technology, one of the popular conversion techniques is limited-range wavelength conversion which can convert input wavelength to some wavelength within a limited range. [12] has shown that four wavelength mixing (FWM) can convert an input wavelength into any output wavelength within 65nm which is the difference between the output and input wavelengths. In [13], connection blocking probability for an all-optical wavelength routing network with FWM wavelength conversion has been investigated with a threshold model in which the FWM wavelength conversion capability is taken into consideration. In [14], first-fit algorithm has been considered for wavelength routing network with FWM wavelength conversion and blocking probability has been derived by layered-graph approach.

With the recent increase of Internet users and the diversity of network applications, QoS provisioning becomes increasingly important in all-optical wavelength routing networks. In [15]–[18], the general approach for service-specific routing and wavelength allocation has been proposed. With the approach, a connection is established according to twofold metrics, i.e., QoS metrics (service requirements) and resource metrics (quality constraints). In this approach, wavelengths are classified into multiple groups which can support different services according to the quality attributes. As for QoS metrics, transmission quality, restoration, network management, and policies have been considered. Given that connections are established according to the above QoS metrics, the connection loss probability of each QoS class has been evaluated.

On the other hand, when wavelengths are transparent to bit rate, protocol, and modulation formats, a connection with any service requirements is established with any idle wavelength [15], [19]. In such a network, QoS guarantee for connection loss probability is also important. Therefore, in this paper, we focus on the connection loss probability as the QoS metric and consider a QoS-guaranteed wavelength allocation for wavelength routing network with limited-range wavelength conversion.

In the proposed allocation, the pre-determined number of wavelengths are allocated to each QoS class depending on the priority of loss probability. Moreover, the wavelength set for the highest priority class includes all wavelengths multiplexed in an optical fiber so as to decrease the connection loss probability. When a connection of a QoS class is established along several links, an idle wavelength in the wave-

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length set of the class is allocated at each link. Here, we consider two wavelength selection rules according to which idle wavelength is selected from the wavelength set for requested QoS class. The connection loss probability of each class greatly depends on the combination of the wavelength selection rules. We consider three combinations of wavelength selection rules and compare those in single link and uni-directional ring network [8], [19].

As for the performance evaluation of the QoS-guaranteed wavelength allocation, we derive connection loss probability of each QoS class on a single link in wavelength routing network using continuous-time Markov chain. With this analytical result, we investigate the impact of three combinations of wavelength selection rules on connection loss probability of each QoS class. We also investigate the connection loss probability for a uni-directional ring network with limited wavelength conversion by simulation.

The rest of the paper is organized as follows. Section 2 represents the QoS-guaranteed wavelength allocation method. In Sect. 3, we present our analytical model on a single link in wavelength routing network and derive the connection loss probability of each QoS class. Numerical examples are shown in Sect. 4 and conclusions are presented in Sect. 5.

## 2. QoS-Guaranteed Wavelength Allocation

In this section, we present our QoS-guaranteed wavelength allocation method in detail. We consider an all-optical wavelength routing network where each node has FWM wavelength conversion. Let  $W$  denote the number of wavelengths multiplexed into an optical fiber. According to [13] and [14], we assume that the range of FWM wavelength conversion for wavelength  $w_i$  ( $1 \leq i \leq W$ ) is from  $w_{\max(1, i-\theta)}$  to  $w_{\min(i+\theta, W)}$  ( $0 \leq \theta \leq W-1$ ) where  $\theta$  is a non-negative integer and called threshold in the following. Note that the FWM wavelength conversions with  $\theta = 0$  and  $W-1$  are corresponding to no wavelength conversion and full-range wavelength conversion, respectively.

In this wavelength routing network,  $M$  QoS classes require different acceptable loss probabilities.  $M$  QoS classes are numbered from 1 to  $M$  and class  $i$  has high priority over class  $j$  when  $i < j$  and the class  $i$  requires smaller connection loss probability than class  $j$ . Therefore, connections of class 1 have the highest priority and require the smallest loss probability.

In our QoS-guaranteed wavelength allocation,  $W$  wavelengths  $\{w_1, \dots, w_W\}$  are classified into  $M$  wavelength sets  $D^{(i)}$  ( $i = 1, \dots, M$ ). Let  $W^{(i)}$  denote the number of wavelengths in  $D^{(i)}$ . Connection of class  $i$  is established with wavelength in  $D^{(i)}$ . Each  $D^{(i)}$  and  $W^{(i)}$  satisfy the followings.

$$D^{(M)} \subset \dots \subset D^{(i)} \subset \dots \subset D^{(1)}, \quad (1)$$

$$D^{(i)} = \{w_1, \dots, w_{W^{(i)}}\}, \quad 1 \leq i \leq M, \quad (2)$$

$$0 < W^{(M)} < \dots < W^{(i)} < \dots < W^{(1)} = W. \quad (3)$$

(3) implies that higher priority class can use more wave-

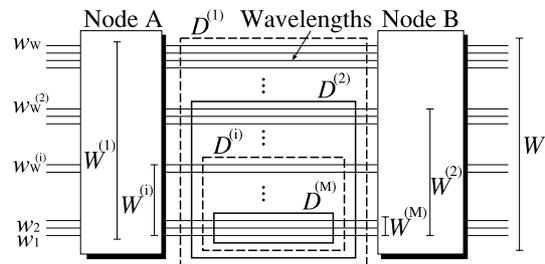


Fig. 1 QoS-guaranteed wavelength allocation.

lengths and it is expected that the resulting connection loss probability of high priority class is small. Figure 1 shows how  $W$  wavelengths are classified into  $M$  QoS classes in the proposed method.

In the QoS-guaranteed wavelength allocation, the following two different rules of wavelength selection are considered.

**Rule 1:** The wavelength with the *minimum* index number in  $D^{(i)}$  is selected.

**Rule 2:** The wavelength with the *maximum* index number in  $D^{(i)}$  is selected.

A connection of class  $i$  is established with an idle wavelength in  $D^{(i)}$  at each link. Each QoS class follows either Rule 1 or Rule 2. The wavelength selection rule of each QoS class affects the performance of the proposed method. Note that the number of available wavelengths for class  $i$  under Rule 1 is likely to be larger than that under Rule 2. In other words, the connection loss probability of class  $i$  under Rule 1 is likely to be smaller than that under Rule 2. However, the connection establishment of class  $i$  under Rule 1 directly affects the number of available wavelengths for lower priority classes than  $i$ . This implies that the traffic intensity of class  $i$  under Rule 1 greatly affects the connection loss probabilities for lower priority classes.

When the class  $i$  follows Rule 2, on the other hand, the connection establishment of class  $i$  does not significantly affect the number of available wavelengths for lower priority classes and this means that the connection loss probabilities of lower priority classes are less affected by the traffic intensity of class  $i$ . Note that the connection establishment under Rule 2 hardly has a large impact on higher priority classes.

Because the number of classes is  $M$ , there are  $2^M$  combinations of the wavelength selection rules. In this paper, however, we consider three combinations shown in Table 1.

In Method 1, all classes follow Rule 1 and, in Method 2, class 1 follows Rule 2 and the other classes follow Rule 1. Classes 1 and 2 follow Rule 2 and other classes follow Rule 1 in Method 3. Note that the number of available wavelengths for lower priority classes than classes 1 and 2 for Method 1 is likely to be the smallest while that for Method 3 the largest.

Here, we explain how a connection of each class is established between end nodes. As mentioned the above,  $W$  wavelengths  $\{w_1, \dots, w_W\}$  are multiplexed into a fiber at

**Table 1** Three combinations of wavelength selection rules.

	class 1	class 2	other classes
Method 1	Rule 1	Rule 1	Rule 1
Method 2	Rule 2	Rule 1	Rule 1
Method 3	Rule 2	Rule 2	Rule 1

every link and each node has an FWM wavelength converter with threshold  $\theta$ . At each link,  $W$  wavelengths are classified into  $M$  wavelength sets and wavelength set  $D^{(i)}$  ( $i = 1, \dots, M$ ) is allocated to class  $i$ . When  $w_j \in D^{(i)}$  is selected for connection of class  $i$  at some link, the conversion range for wavelength at the next link is from  $w_{\max(1, j-\theta)}$  to  $w_{\min(j+\theta, W^{(i)})}$ . In this case, an available wavelength for the next link is selected according to either of the following two procedures based on first-fit algorithm [14].

**Procedure 1:** If class  $i$  ( $i = 1, \dots, M$ ) follows Rule 1, an idle wavelength with the *minimum* index number in the set  $\{w_{\max(1, j-\theta)}, \dots, w_{\min(j+\theta, W^{(i)})}\}$  is selected.

**Procedure 2:** If class  $i$  ( $i = 1, \dots, M$ ) follows Rule 2, an idle wavelength with the *maximum* index number in the set  $\{w_{\max(1, j-\theta)}, \dots, w_{\min(j+\theta, W^{(i)})}\}$  is selected.

If wavelength allocations in all links along the path succeed, lightpath connection is eventually established.

### 3. Performance Analysis

In this section, we derive the connection loss probability of each QoS class for a single link in wavelength routing network. We use the following assumptions.

1.  $W$  wavelengths are multiplexed into a fiber at a single link.
2. The number of QoS classes is  $M$  and class  $i$  ( $i = 1, \dots, M$ ) has priority over class  $j$  if  $i < j$ .
3. Connections of class  $i$  arrive at the single link according to a Poisson process with rate  $\lambda^{(i)}$  and total arrival rate is  $\lambda = \sum_{i=1}^M \lambda^{(i)}$ .
4. Connection holding times of all classes are exponentially distributed with rate  $\mu$ .
5. No queueing for connection request is permitted, that is, the connection is lost immediately after the connection establishment fails.

Let  $B^{(i)}$  ( $i = 1, \dots, M$ ) denote the wavelength set given by

$$B^{(i)} = \begin{cases} D^{(i)} - D^{(i+1)}, & i < M, \\ D^{(M)}, & i = M, \end{cases} \quad (4)$$

where  $D^{(i)}$  is a wavelength set of class  $i$ . In addition, we define  $\bar{W}^{(i)}$  ( $i = 1, \dots, M$ ) as the number of wavelengths in  $B^{(i)}$ . We have

$$\bar{W}^{(i)} = \begin{cases} W^{(i)} - W^{(i+1)}, & i < M, \\ W^{(M)}, & i = M. \end{cases} \quad (5)$$

Let  $N^{(i)}(t)$  ( $i = 1, \dots, M$ ) denote the number of wavelengths which are utilized in  $B^{(i)}$  at time  $t$ . Note that

$$0 \leq N^{(i)}(t) \leq \bar{W}^{(i)}, \quad i = 1, \dots, M. \quad (6)$$

We define the state of the link at time  $t$  as

$$(N^{(1)}(t), \dots, N^{(i)}(t), \dots, N^{(M)}(t)). \quad (7)$$

Let  $U$  denote the state space of  $(N^{(1)}(t), \dots, N^{(M)}(t))$ . From the above assumptions,  $(N^{(1)}(t), \dots, N^{(M)}(t))$  is a continuous-time Markov chain [20]. Since we consider the queueing behavior in equilibrium, we omit  $t$  in the following. In Tables 2, 3 and 4, we show transition rates from the state  $(N^{(1)}, \dots, N^{(i)}, \dots, N^{(M)})$  in Methods 1, 2 and 3, respectively.

Let  $\pi(N^{(1)}, \dots, N^{(M)})$  denote the steady state probability of  $(N^{(1)}, \dots, N^{(M)})$ .  $\pi(N^{(1)}, \dots, N^{(M)})$  is uniquely determined by equilibrium state equations and following normalized condition

$$\sum_{(N^{(1)}, \dots, N^{(M)}) \in U} \pi(N^{(1)}, \dots, N^{(M)}) = 1. \quad (8)$$

Equilibrium state equations for Method 2 are shown in Appendix. Similarly, those for other methods can be obtained from Tables 2 and 4.

With  $\pi(N^{(1)}, \dots, N^{(M)})$ , connection loss probability of class  $i$ ,  $P_{loss}^{(i)}$ , is given by

$$P_{loss}^{(i)} = \sum_{(N^{(1)}, \dots, N^{(i-1)}) \in U^{(i-1)}} \pi(N^{(1)}, \dots, N^{(i-2)}, N^{(i-1)}, \bar{W}^{(i)}, \bar{W}^{(i+1)}, \dots, \bar{W}^{(M)}). \quad (9)$$

Here,  $U^{(i)}$  denotes the state space of  $(N^{(1)}, \dots, N^{(i)})$  and  $\bar{U}^{(i)}$  the state space of  $(N^{(i)}, \dots, N^{(M)})$ .

### 4. Numerical Examples

In this section, we show some numerical examples for the QoS-guaranteed wavelength allocation in cases of Methods 1, 2 and 3. First we consider a single link in wavelength routing network, and then we consider a uni-directional ring network. In both cases, we assume that the number of QoS classes is three. Moreover, we assume that the connection holding time is exponentially distributed with rate  $\mu = 1$ .

#### 4.1 Single Link in Wavelength Routing Network

In this subsection, we consider a single link in wavelength routing network. The connection loss probabilities of three QoS classes,  $P_{loss}^{(1)}$ ,  $P_{loss}^{(2)}$ , and  $P_{loss}^{(3)}$ , are calculated by the analysis in the previous section and by simulation.

##### 4.1.1 Impact of Total Connection Arrival Rate

First, we consider how the total arrival rate of connections affects connection loss probability for each QoS class. Here we assume that the number of wavelengths is  $W = 32$ . 32 wavelengths are classified into  $D^{(1)}$ ,  $D^{(2)}$  and  $D^{(3)}$  and the numbers of wavelengths in these sets are given by  $W^{(1)} = 32$ ,  $W^{(2)} = 16$  and  $W^{(3)} = 10$ , respectively. In addition, we

**Table 2** State transition rate in Method 1.

Current state: $(N^{(1)}, \dots, N^{(i)}, \dots, N^{(M)})$	Next state	Transition rate
$N^{(M)} < \bar{W}^{(M)}$	$(N^{(1)}, \dots, N^{(M)} + 1)$	$\lambda$
$N^{(i)} < \bar{W}^{(i)}$ for $\exists i \in \{1, \dots, M-1\}$ , $N^{(k)} = \bar{W}^{(k)}$ for $\forall k \in \{i+1, \dots, M\}$	$(N^{(1)}, \dots, N^{(i)} + 1, \dots, N^{(M)})$	$\sum_{m=1}^i \lambda^{(m)}$
$N^{(i)} > 0$	$(N^{(1)}, \dots, N^{(i)} - 1, N^{(M)})$	$N^{(i)}\mu$

**Table 3** State transition rate in Method 2.

Current state: $(N^{(1)}, \dots, N^{(i)}, \dots, N^{(M)})$	Next state	Transition rate
$N^{(1)} < \bar{W}^{(1)}$	$(N^{(1)} + 1, \dots, N^{(M)})$	$\lambda^{(1)}$
$N^{(i)} < \bar{W}^{(i)}$ for $\exists i \in \{2, \dots, M-1\}$ , $N^{(j)} = \bar{W}^{(j)}$ for $\forall j \in \{1, \dots, i-1\}$ , $N^{(k)} < \bar{W}^{(k)}$ for $\exists k \in \{i+1, \dots, M\}$	$(N^{(1)}, \dots, N^{(i)} + 1, \dots, N^{(M)})$	$\lambda^{(1)}$
$N^{(M)} < \bar{W}^{(M)}$ , $N^{(i)} < \bar{W}^{(i)}$ for $\exists i \in \{1, \dots, M-1\}$	$(N^{(1)}, \dots, N^{(M)} + 1)$	$\sum_{m=2}^M \lambda^{(m)}$
$N^{(i)} < \bar{W}^{(i)}$ for $\exists i \in \{2, \dots, M-1\}$ , $N^{(j)} < \bar{W}^{(j)}$ for $\exists j \in \{1, \dots, i-1\}$ , $N^{(k)} = \bar{W}^{(k)}$ for $\forall k \in \{i+1, \dots, M\}$	$(N^{(1)}, \dots, N^{(i)} + 1, \dots, N^{(M)})$	$\sum_{m=2}^i \lambda^{(m)}$
$N^{(M)} < \bar{W}^{(M)}$ , $N^{(i)} = \bar{W}^{(i)}$ for $\forall i \in \{1, \dots, M-1\}$	$(N^{(1)}, \dots, N^{(M)} + 1)$	$\lambda$
$N^{(i)} < \bar{W}^{(i)}$ for $\exists i \in \{2, \dots, M-1\}$ , $N^{(j)} = \bar{W}^{(j)}$ for $\forall j \in \{1, \dots, i-1\}$ $N^{(k)} = \bar{W}^{(k)}$ for $\forall k \in \{i+1, \dots, M\}$	$(N^{(1)}, \dots, N^{(i)} + 1, \dots, N^{(M)})$	$\sum_{m=1}^i \lambda^{(m)}$
$N^{(i)} > 0$	$(N^{(1)}, \dots, N^{(i)} - 1, \dots, N^{(M)})$	$N^{(i)}\mu$

**Table 4** State transition rate in Method 3.

Current state: $(N^{(1)}, \dots, N^{(i)}, \dots, N^{(M)})$	Next state	Transition rate
$N^{(1)} < \bar{W}^{(1)}$	$(N^{(1)} + 1, \dots, N^{(M)})$	$\lambda^{(1)}$
$N^{(1)} < \bar{W}^{(1)}$ , $N^{(2)} < \bar{W}^{(2)}$	$(N^{(1)}, N^{(2)} + 1, \dots, N^{(M)})$	$\lambda^{(2)}$
$N^{(i)} < \bar{W}^{(i)}$ , $N^{(i)} < \bar{W}^{(i)}$ for $\exists i \in \{3, \dots, M-1\}$ , $N^{(j)} = \bar{W}^{(j)}$ for $\forall j \in \{2, \dots, i-1\}$ , $N^{(k)} < \bar{W}^{(k)}$ for $\exists k \in \{i+1, \dots, M\}$	$(N^{(1)}, \dots, N^{(i)} + 1, \dots, N^{(M)})$	$\lambda^{(2)}$
$N^{(M)} < \bar{W}^{(M)}$ , $N^{(i)} < \bar{W}^{(i)}$ for $\exists i \in \{2, \dots, M-1\}$	$(N^{(1)}, \dots, N^{(M)} + 1)$	$\sum_{m=3}^M \lambda^{(m)}$
$N^{(i)} < \bar{W}^{(i)}$ for $\exists i \in \{3, \dots, M-1\}$ , $N^{(j)} < \bar{W}^{(j)}$ for $\exists j \in \{2, \dots, i-1\}$ , $N^{(k)} = \bar{W}^{(k)}$ for $\forall k \in \{i+1, \dots, M\}$	$(N^{(1)}, \dots, N^{(i)} + 1, \dots, N^{(M)})$	$\sum_{m=3}^i \lambda^{(m)}$
$N^{(i)} < \bar{W}^{(i)}$ for $\exists i \in \{2, \dots, M-1\}$ , $N^{(j)} = \bar{W}^{(j)}$ for $\forall j \in \{1, \dots, i-1\}$ , $N^{(k)} < \bar{W}^{(k)}$ for $\exists k \in \{i+1, \dots, M\}$	$(N^{(1)}, \dots, N^{(i)} + 1, \dots, N^{(M)})$	$\lambda^{(1)} + \lambda^{(2)}$
$N^{(M)} < \bar{W}^{(M)}$ , $N^{(1)} < \bar{W}^{(1)}$ $N^{(j)} = \bar{W}^{(j)}$ for $\forall j \in \{2, \dots, M-1\}$	$(N^{(1)}, \dots, N^{(M)} + 1)$	$\sum_{m=2}^M \lambda^{(m)}$
$N^{(i)} < \bar{W}^{(i)}$ for $\exists i \in \{2, \dots, M-1\}$ , $N^{(1)} < \bar{W}^{(1)}$ , $N^{(j)} = \bar{W}^{(j)}$ for $\forall j \in \{2, \dots, i-1\}$ $N^{(k)} = \bar{W}^{(k)}$ for $\forall k \in \{i+1, \dots, M\}$	$(N^{(1)}, \dots, N^{(i)} + 1, \dots, N^{(M)})$	$\sum_{m=2}^i \lambda^{(m)}$
$N^{(M)} < \bar{W}^{(M)}$ $N^{(j)} = \bar{W}^{(j)}$ for $\forall j \in \{1, \dots, M-1\}$	$(N^{(1)}, \dots, N^{(M)} + 1)$	$\lambda$
$N^{(i)} < \bar{W}^{(i)}$ for $\exists i \in \{3, \dots, M-1\}$ , $N^{(j)} = \bar{W}^{(j)}$ for $\forall j \in \{1, \dots, i-1\}$ $N^{(k)} = \bar{W}^{(k)}$ for $\forall k \in \{i+1, \dots, M\}$	$(N^{(1)}, \dots, N^{(i)} + 1, \dots, N^{(M)})$	$\sum_{m=1}^i \lambda^{(m)}$
$N^{(i)} > 0$	$(N^{(1)}, \dots, N^{(i)} - 1, \dots, N^{(M)})$	$N^{(i)}\mu$

set  $\lambda^{(1)} = \lambda^{(2)} = \lambda^{(3)} = \lambda/3$ .

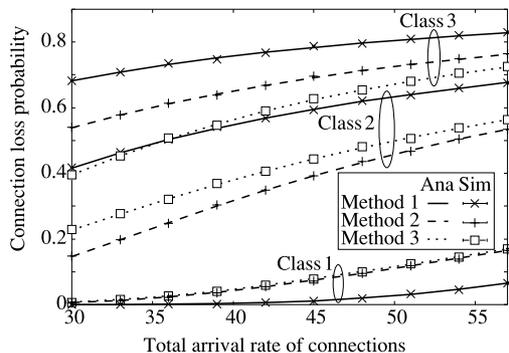
In Fig. 2, lines and dots denote the results of the analysis and simulation, respectively. From this figure, we observe that analytical and simulation results are almost the same regardless of the increase of total arrival rate. Therefore the analytical results are effective for the evaluation of three methods under the above setting.

We also see that the QoS-guaranteed wavelength allocation method provides multiple QoS classes in terms of the connection loss probability. The connection loss probability of class 1 for any method is the smallest among three prior-

ity classes because connections of class 1 can utilize more wavelengths than those of the other classes. However, this results in the large loss probabilities of classes 2 and 3.

As for the effect of the combination of wavelength selection rules, the loss probability of class 1 for Method 1 is the smallest among three Methods. This is because for Method 1, the connections of class 1 are likely to utilize the largest number of wavelengths in  $D^{(2)}$  and  $D^{(3)}$  among three methods.

We also observe from this figure that the connection loss probability for any method increases as the total con-



**Fig. 2** Connection loss probability vs. total connection arrival rate for a single link.

nection arrival rate becomes large. Nevertheless, for each QoS class, the above tendency of connection loss probabilities for three methods does not change.

#### 4.1.2 Impact of the Loss Probability Required for Each QoS Class

Next, we consider how the connection loss probability required for each QoS class affects the performances of Methods 1, 2, and 3. Here we assume that the number of wavelengths  $W = 32$  and that  $\lambda^{(1)} = \lambda^{(2)} = \lambda^{(3)} = 7$ . In addition, it is required that  $P_{loss}^{(3)}$  is smaller than or equal to the constant  $\alpha$ . When  $\alpha$  is given, the number of wavelengths for each QoS class is determined so as to satisfy  $P_{loss}^{(3)} \leq \alpha$ . Note that as  $\alpha$  becomes small, the number of wavelengths for each QoS class is restricted to a small set.

Connection loss probabilities  $P_{loss}^{(1)}$ ,  $P_{loss}^{(2)}$  and  $P_{loss}^{(3)}$  are calculated with (9) for all  $(W^{(2)}, W^{(3)})$ 's such that  $0 < W^{(3)} < W^{(2)} < 32$ , and  $P_{loss}^{(1)}$  and  $P_{loss}^{(2)}$  with which  $P_{loss}^{(3)} \leq \alpha$  holds are plotted in Fig. 3. We also calculate with M/M/c/c the connection loss probability for single QoS class where no QoS is guaranteed (no QoS in Fig. 3).

Table 5 shows the comparison of analytical results with simulation ones (with 95% confidence interval). From this table, we find that those results are almost the same regardless of QoS class and method in the case of  $W^{(1)} = 32$ ,  $W^{(2)} = 25$  and  $W^{(3)} = 23$ . We have investigated other cases of  $(W^{(2)}, W^{(3)})$ 's and observed that analytical results are almost the same as simulation ones. Therefore, our analytical results under the traffic condition  $\lambda^{(i)} = 7$  for  $i = 1, 2$  and 3 are efficient enough to discuss the performance of the proposed method.

Figures 3(a), (b), and (c) show  $P_{loss}^{(1)}$  and  $P_{loss}^{(2)}$  for Methods 1, 2, and 3 in the case of  $\alpha = 1.0$ , respectively. Note that, in the case with  $\alpha = 1.0$ , any combination of  $(W^{(2)}, W^{(3)})$  satisfies  $P_{loss}^{(3)} \leq \alpha$ . From these figures, we observe that Method 1 can provide the smallest loss probability for class 1 among three methods. This is because, in Method 1, all QoS classes follow the wavelength selection rule 1 and connections of class 1 can use more wavelengths in  $D^{(2)}$  and  $D^{(3)}$ . However, Method 1 tends to provide larger loss probability for class 2 than Methods 2 and 3. On the other hand, in Method

**Table 5** Comparison of analytical results with simulation ones (with 95% confidence interval) in the case of  $W^{(2)} = 25$  and  $W^{(3)} = 23$ .

(a) Method 1.		
	Analysis	Simulation
$P_{loss}^{(1)}$	7.418504e-05	7.289973e-05±1.067724e-05
$P_{loss}^{(2)}$	5.388198e-02	5.413990e-02±0.042510e-02
$P_{loss}^{(3)}$	1.055443e-01	1.057769e-01±0.005052e-01

(b) Method 2.		
	Analysis	Simulation
$P_{loss}^{(1)}$	3.852461e-03	3.802187e-03±0.113577e-03
$P_{loss}^{(2)}$	8.275647e-03	8.224171e-03±0.166026e-03
$P_{loss}^{(3)}$	1.400443e-02	1.400785e-02±0.023819e-01

(c) Method 3.		
	Analysis	Simulation
$P_{loss}^{(1)}$	4.095847e-03	4.045786e-03±0.113599e-03
$P_{loss}^{(2)}$	9.383536e-03	9.346767e-03±0.179258e-03
$P_{loss}^{(3)}$	1.040093e-02	1.034486e-02±0.019668e-02

2, class 1 follows the rule 1 and classes 2 and 3 follow the rule 2. Because connections of class 2 can use more wavelengths in  $D^{(2)}$  and  $D^{(3)}$ , Method 2 can provide the smallest loss probability for class 2 among three methods.

Figures 3(d), (e), and (f) show  $P_{loss}^{(1)}$  and  $P_{loss}^{(2)}$  in the case of  $\alpha = 0.5$ . As is the case with  $\alpha = 1.0$ , we observe from Figs. 3(d) and (e) that Method 1 can provide the smallest connection loss probability for class 1, and that Method 2 can provide the smallest connection loss probability for class 2. With Method 2, both loss probabilities of classes 1 and 2 can be smaller than the connection loss probability provided in no QoS-guaranteed network. Because, in Method 1, classes 1 and 2 adopt Rule 1, the number of wavelengths which class 2 can use becomes slightly large and this results in the slight decrease of  $P_{loss}^{(2)}$ . That is, in Method 1, increasing  $W^{(2)}$  does not improve  $P_{loss}^{(2)}$  so much. On the other hand, in Method 2,  $P_{loss}^{(2)}$  is greatly improved by the increase of  $W^{(2)}$ . This implies that the improvement of  $P_{loss}^{(2)}$  depends on not only  $W^{(2)}$  but also the rule adopted by individual QoS class.

Figures 3(g), (h), and (i) show the case of  $\alpha = 0.05$ . As  $\alpha$  becomes small, connection loss probabilities for classes 1 and 2 become large and the number of pairs of  $P_{loss}^{(1)}$  and  $P_{loss}^{(2)}$  which satisfy  $P_{loss}^{(3)} \leq \alpha$  becomes small. This is because the number of wavelengths allocated for class 3 increases and thus causes the decrease of wavelengths available for classes 1 and 2.

Figures 3(j), (k) and (l) show  $P_{loss}^{(1)}$  and  $P_{loss}^{(2)}$  in the case of  $\alpha = 0.01$ . We observe that Methods 2 and 3 can provide a small number of pairs of  $P_{loss}^{(1)}$  and  $P_{loss}^{(2)}$  while no wavelength combination exists for Method 1. In Method 1, all QoS classes follow Rule 1 and the resulting number of wavelengths utilized by class 3 decreases. On the other hand, Rule 2 is adopted for class 1 in Method 2 and classes 1 and 2 in Method 3, and this causes the increase of the number of wavelengths utilized by class 3.

Furthermore, Method 3 can provide 46 pairs of  $P_{loss}^{(1)}$  and  $P_{loss}^{(2)}$  whereas Method 2 provides 26 pairs. In Method 3, only class 3 follows the Rule 1. Hence, with Method

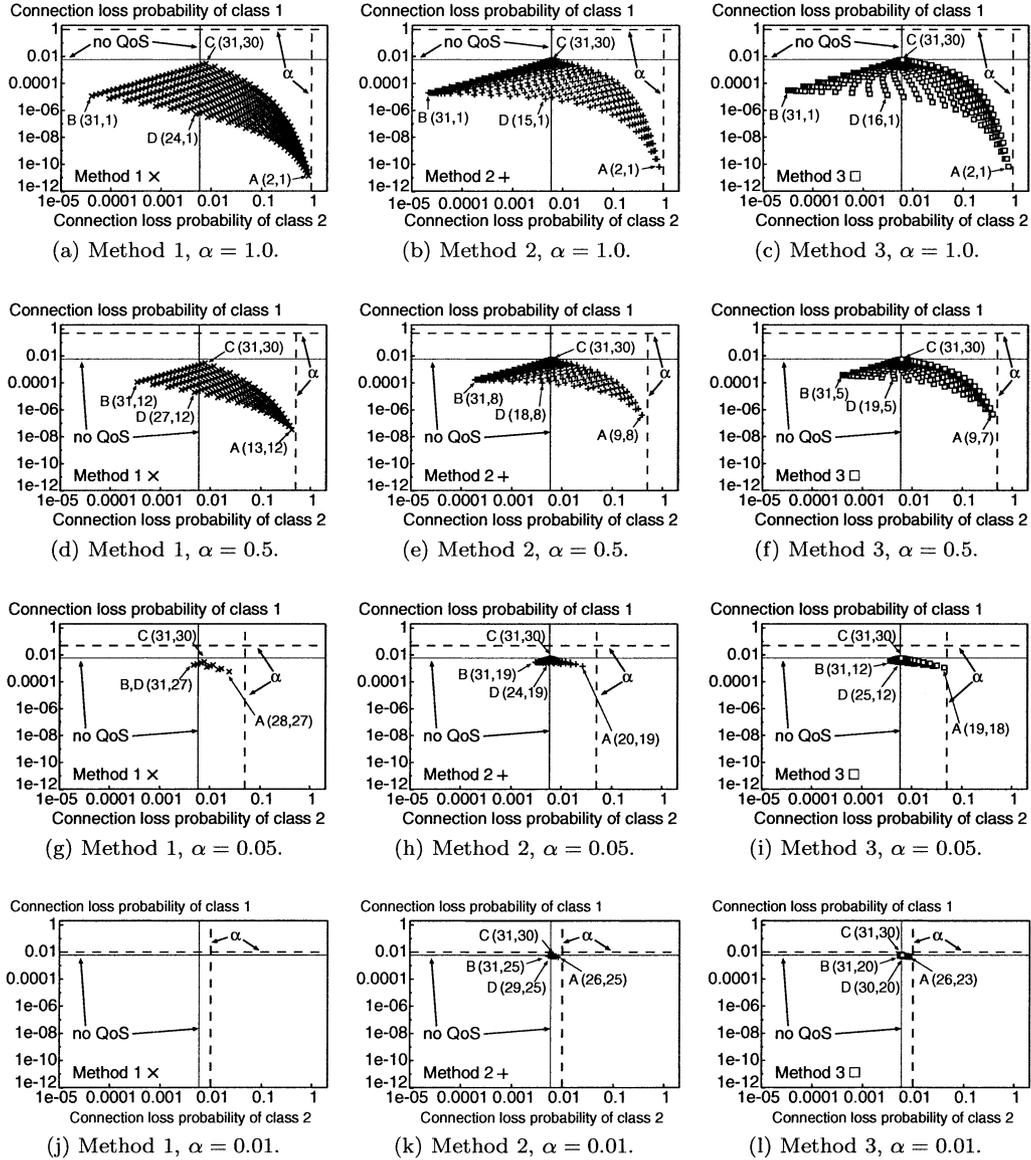


Fig. 3 Impact of required loss probability for each QoS class.

3, connections of class 3 can use more wavelengths in  $D^{(3)}$  and Method 3 can provide the smallest loss probability for the lowest priority class 3 among three methods. Therefore, Method 3 is effective when low priority class requires small connection loss probability.

In Fig. 3, the following four points are defined.

- A:  $P_{loss}^{(1)}$  is the smallest.
- B:  $P_{loss}^{(2)}$  is the smallest.
- C:  $P_{loss}^{(3)}$  is the smallest.
- D:  $P_{loss}^{(1)}$  is the smallest among the points such that both  $P_{loss}^{(1)}$  and  $P_{loss}^{(2)}$  are smaller than the connection loss probability in the no QoS guaranteed case.

We also present  $(W^{(2)}, W^{(3)})$  at each point.

When  $\alpha$  is one, regardless of methods,  $(W^{(2)}, W^{(3)}) =$

$(2, 1)$ ,  $(31, 1)$ , and  $(31, 30)$  provide the smallest loss probability for classes 1, 2 and 3, respectively. As  $\alpha$  decreases, larger  $W^{(3)}$  is required to provide the smaller connection loss probability of class 3. In particular, Method 1 requires the largest  $W^{(3)}$  among three methods.

In Methods 1 and 2, the point A is given by  $(W^{(2)}, W^{(3)}) = (W^{(3)} + 1, W^{(3)})$ , as expected. However, A in Method 3 does not always satisfy  $(W^{(3)} + 1, W^{(3)})$  (see Figs. 3(f) and (l)). This is because  $P_{loss}^{(3)}$  for Method 3 is largely affected by  $\bar{W}^{(2)}$ . When  $\bar{W}^{(2)}$  is small, class 2 is likely to use wavelength in  $D^{(3)}$  and this causes large  $P_{loss}^{(3)}$ . As a result,  $P_{loss}^{(3)} \leq \alpha$  does not hold with  $(W^{(2)}, W^{(3)}) = (W^{(3)} + 1, W^{(3)})$ .

As for the point B and C, we always have  $W^{(2)} = W - 1$  for B and  $(W^{(2)}, W^{(3)}) = (W - 1, W - 2)$  for C.

From Fig. 3, we can obtain the best wavelength alloca-

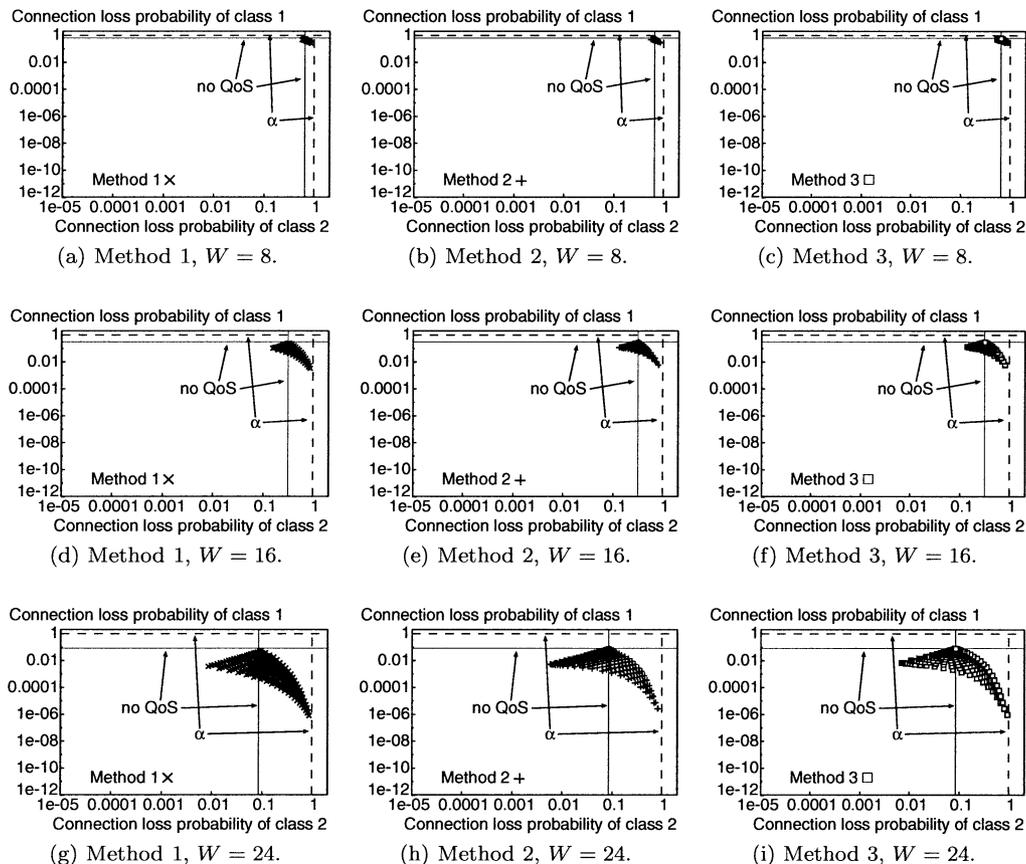


Fig. 4 Impact of the number of wavelengths.

tion according to a given QoS policy. For example, if we have a QoS policy in which  $P_{loss}^{(1)}$  and  $P_{loss}^{(2)}$  are smaller than the connection loss probability and  $P_{loss}^{(3)} \leq \alpha$ , the point D provides the best combination of  $(W^{(2)}, W^{(3)})$ .

#### 4.1.3 Impact of the Number of Wavelengths

We consider how the number of wavelengths affects the performance of QoS-guaranteed wavelength allocation. Here we assume that  $\lambda^{(1)} = \lambda^{(2)} = \lambda^{(3)} = 7$ .

Figure 4 shows  $P_{loss}^{(1)}$  and  $P_{loss}^{(2)}$  when  $P_{loss}^{(3)} \leq \alpha = 1.0$ . Figures 4(a), (b), and (c), Figs. 4(d), (e), and (f), and Figs. 4(g), (h), and (i) show the results in the cases of  $W = 8$ , 16, and 24, respectively. Note that Figs. 3(a), (b) and (c) correspond to the results in the case of  $W = 32$ .

From Figs. 4(a), (b), and (c), we observe that Methods 1, 2, and 3 have almost the same performance when the number of wavelengths is eight. This is because  $\bar{W}^{(1)}$ ,  $\bar{W}^{(2)}$  and  $\bar{W}^{(3)}$  are small and the wavelength selection rules do not affect the connection loss probability of each class. Even if the number of wavelengths becomes 16, we can see the same tendency from Figs. 4(d), (e), and (f).

When the number of wavelengths becomes 24 (see Figs. 4(g), (h), and (i)), the connection loss probability of each class comes to depend on wavelength selection rule. Method 1 can provide the smallest connection loss prob-

ity for class 1 among three method because the connection of class 1 can use more wavelengths in  $D^{(3)}$ . Methods 2 and 3 can provide smaller connection loss probability for class 2 than Method 1 because the connection of class 2 can use more wavelengths in  $D^{(3)}$ . However, Method 2 can provide smaller connection loss probability for class 1 than Method 3 because class 1 can use more wavelengths in  $\bar{D}^{(2)}$ . From the above, when the number of wavelengths is large, it is important to adopt a suitable method for QoS provisioning policy.

#### 4.2 Ring Network

In this subsection, we investigate the performance of the proposed method in a uni-directional ring network. Here, in the uni-directional ring network, the benefit of wavelength conversion is limited due to load correlation [8]. If  $\theta$  becomes large, the performances of three methods do not change so much. Therefore, the results similar to those in the ring network are obtained in other network topologies where the benefit of wavelength conversion is large<sup>†</sup>.

In the ring network, in addition to the assumptions in Sect. 3, we assume that the number of nodes  $L$  is equal to 10

<sup>†</sup>We investigated the performance of the proposed method in a random mesh network and obtained the tendency similar to those in the ring network.

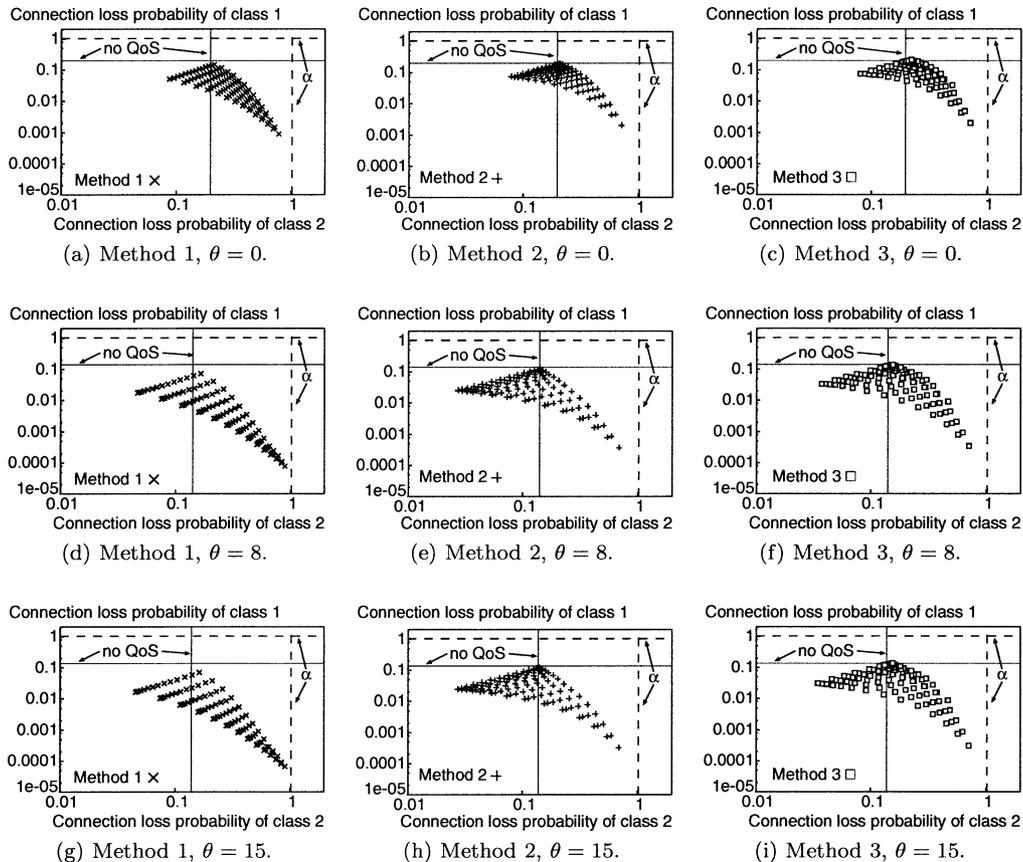


Fig. 5 Impact of threshold.

and that the number of wavelengths  $W$  equals 32. Moreover, we assume that all nodes have the capability of FWM wavelength conversion with threshold  $\theta$ . The pair of source and destination nodes of a connection is distributed uniformly, i.e., any pair is selected with the same probability. In the case of ring network, the connection loss probability is calculated by simulation. We also evaluate the connection loss probability in no QoS-guaranteed network with FWM wavelength conversion by simulation.

#### 4.2.1 Impact of Threshold

In this subsection, we consider how the threshold of FWM wavelength conversion affects the performances of Methods 1, 2, and 3. Here we assume that the number of wavelengths  $W = 16$  and that  $\lambda^{(1)} = \lambda^{(2)} = \lambda^{(3)} = 9$ .

As is the case with Fig. 4, Fig. 5 shows  $P_{loss}^{(1)}$  and  $P_{loss}^{(2)}$  when  $P_{loss}^{(3)} \leq \alpha = 1.0$ . Figures 5(a), (b), and (c), Figs. 5(d), (e), and (f), and Figs. 5(g), (h), and (i) show the simulation results in the cases of  $\theta = 0, 8$ , and 15, respectively.

From Figs. 5(a), (b), and (c), we can find that the performances of three methods are almost the same. Note that  $\theta = 0$  corresponds to no wavelength conversion and this causes the large connection loss probability of each QoS class. In this case, no matter what method is adopted, the connection loss probability of each QoS class does not change so much.

When  $\theta$  is 8 (see Figs. 5(d), (e), and (f)), Method 1 provides the smallest connection loss probability for class 1 and Method 2 provides the smallest connection loss probability for class 2, as expected. On the other hand, Method 3 can not provide the smallest connection loss probability for both classes 1 and 2.

We also observe the tendency in Figs. 5(g), (h), and (i), those are the case of  $\theta = 15$ . Comparing the case of  $\theta = 8$  with that of  $\theta = 15$ , we observe that the results are almost the same in each method. Note that  $\theta = 15$  corresponds to full-range wavelength conversion for each wavelength set. These figures show that large conversion capability does not always improve connection loss probability remarkably.

We have also investigated the effect of the wavelength conversion capability on the connection loss probability. Figure 6 shows how the threshold affects the connection loss probability for three methods. We assume that  $W = 32$  and  $\lambda^{(1)} = \lambda^{(2)} = \lambda^{(3)} = 20$ . We set  $W^{(1)} = 32$ ,  $W^{(2)} = 19$ , and  $W^{(3)} = 7$  for Method 1,  $W^{(1)} = 32$ ,  $W^{(2)} = 10$ , and  $W^{(3)} = 3$  for Method 2, and  $W^{(1)} = 32$ ,  $W^{(2)} = 10$ , and  $W^{(3)} = 5$  for Method 3 so that connections of class 1 for three Methods have almost the same loss probability when the threshold is equal to zero.

From Fig. 6, we observe that large  $\theta$  is effective for the connection loss probability of class 1 while it does not improve those of classes 2 and 3. In addition, the connection loss probability of class 1 in Method 1 is greatly improved

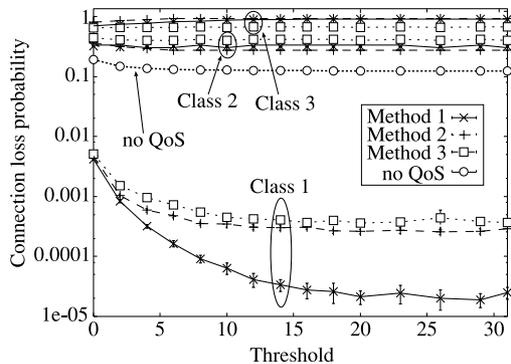


Fig. 6 Connection loss probability vs. threshold for ring network.

by  $\theta$ . Note that the connection loss probability of class 1 in any Method is less improved when  $\theta > 15$ . This suggests that large capability of wavelength conversion is not needed for the improvement of connection loss probability.

#### 4.2.2 Impact of Arrival Rate of Each QoS Class

Finally, we investigate how the arrival rate of each class affects the loss probability of class 1 in ring network. Figures 7(a), (b) and (c), Figs. 7(d), (e) and (f), Figs. 7(g), (h) and (i), and Figs. 7(j), (k) and (l) are the cases of  $\theta = 31$ , 15, 10, and 0, respectively. When  $\lambda^{(i)}$  ( $i = 1, 2$ , and 3) is a variable parameter,  $\lambda^{(j)}$ 's ( $j \neq i$ ) are constant and equal to 20. We set  $W^{(1)} = 32$ ,  $W^{(2)} = 20$  and  $W^{(3)} = 10$  for Method 1,  $W^{(1)} = 32$ ,  $W^{(2)} = 8$  and  $W^{(3)} = 4$  for Method 2, and  $W^{(1)} = 32$ ,  $W^{(2)} = 9$  and  $W^{(3)} = 3$  for Method 3. In the above setting, when the arrival rates of all classes are 20 and the threshold is equal to zero, the loss probabilities of classes 2 and 3 for three methods become almost the same.

From Fig. 7(a), we observe that the connection loss probabilities of class 1 for three methods show the same tendency in the case of  $\theta = 31$ . On the other hand, in Fig. 7(b), the connection loss probability for Method 1 increases as the arrival rate of class 2 becomes large. However, loss probabilities for Methods 2 and 3 are almost constant when the arrival rate of class 2 is larger than 10.

Because Method 1 adopts the wavelength selection rule 1 for all classes, connections of class 1 can use more wavelengths in  $D^{(2)}$  and  $D^{(3)}$  than classes 2 and 3. On the other hand, since Methods 2 and 3 adopt the wavelength rule 2 for class 1, connections of class 1 use less wavelengths in  $D^{(2)}$  and  $D^{(3)}$ . Note that  $W^{(2)}$  and  $W^{(3)}$  for Method 1 is larger than those for Methods 2 and 3. That is, the number of wavelengths which only class 1 can use for Method 1 is smaller than those for Methods 2 and 3. When  $\lambda^{(i)}$  ( $i = 2, 3$ ) is larger than 20, wavelengths in  $D^{(2)}$  and  $D^{(3)}$  are likely to be utilized by classes 2 and 3. In this situation, class 1 is likely to use  $\bar{W}^{(1)}$  wavelengths and this results in the increase of the connection loss probability of class 1 for Method 1.

From Fig. 7(c), we observe that the connection loss probability of class 1 for each method is not affected by the arrival rate of class 3. Note that in each method,  $W^{(3)}$  is smaller than  $W^{(1)} - W^{(3)}$ , that is,  $W^{(3)} : W^{(1)} - W^{(3)} = 10 : 22$

for Method 1, 4 : 28 for Method 2 and 3 : 29 for Method 3. Class 3 can not use  $W^{(1)} - W^{(3)}$  wavelengths and this results in small connection loss probability of class 1 against the increase of class 3 arrivals. Therefore, Methods 2 and 3 are robust in the sense of keeping the connection loss probability of class 1 constant despite the increase of arrival rate of the other classes.

When  $\theta$  decreases from 31 to 15, connection loss probabilities of all classes become large as shown in Figs. 7(d), (e) and (f). This is because the number of available wavelengths on the next link is restricted. From these figures, we also find that the connection loss probability for Method 1 increases as the arrival rate of class 2 becomes large. The connection loss probability of class 1 for Method 1 becomes larger than one for Method 2 when  $\lambda^{(2)}$  is larger than 25.

Figures 7(g), (h) and (i) show the case of  $\theta = 10$ . The connection loss probability of class 1 for Method 1 becomes larger than that for Method 2 when  $\lambda^{(2)}$  is larger than 20. In addition, from Fig. 7(i), we can find that the connection loss probability of class 1 increases as the connection arrival rate of class 3 increases.

Figures 7(j), (k), and (l) show the connection loss probability in ring network without wavelength conversion capability. In these figures, the connection loss probabilities of class 1 for three methods have the same tendency regardless of the connection arrival rate of class 1. However, the connection loss probability of class 1 for Method 1 becomes large as the arrival rate of class 2 or class 3 increases.

From Figs. 7(b), (e), (h), and (k), we find that the advantage of Method 1 decreases as  $\theta$  becomes small. When  $\theta$  is small, idle wavelengths are not used efficiently due to the restriction of wavelength conversion. To decrease the connection loss probability in this case, more wavelengths are required. In Methods 2 and 3, class 1 can use  $W^{(1)} - W^{(2)} = 24$  and 23 wavelengths, respectively. However, in Method 1, class 1 uses only  $W^{(1)} - W^{(2)} = 12$  wavelengths. Therefore, connection loss probability of class 1 for Method 1 increases as  $\theta$  becomes small.

Hence, in the uni-directional ring network, the connection loss probability of class 1 for Method 1 is greatly affected by the arrival rates of lower priority classes. When the wavelength conversion capability in ring network is small, Methods 2 and 3 are more robust and effective than Method 1.

## 5. Conclusions

In this paper, we have proposed a QoS-guaranteed wavelength allocation method which provides multiple QoS classes for the connection loss probability. We have considered three combinations of wavelength selection rules and have compared those performances for a single link and a uni-directional ring network. Numerical results have shown that our analysis is useful for both the optimal allocation of wavelengths and the best selection of Method.

In numerical examples, we have found that Method 1 is effective to the highest priority class. However, the connec-

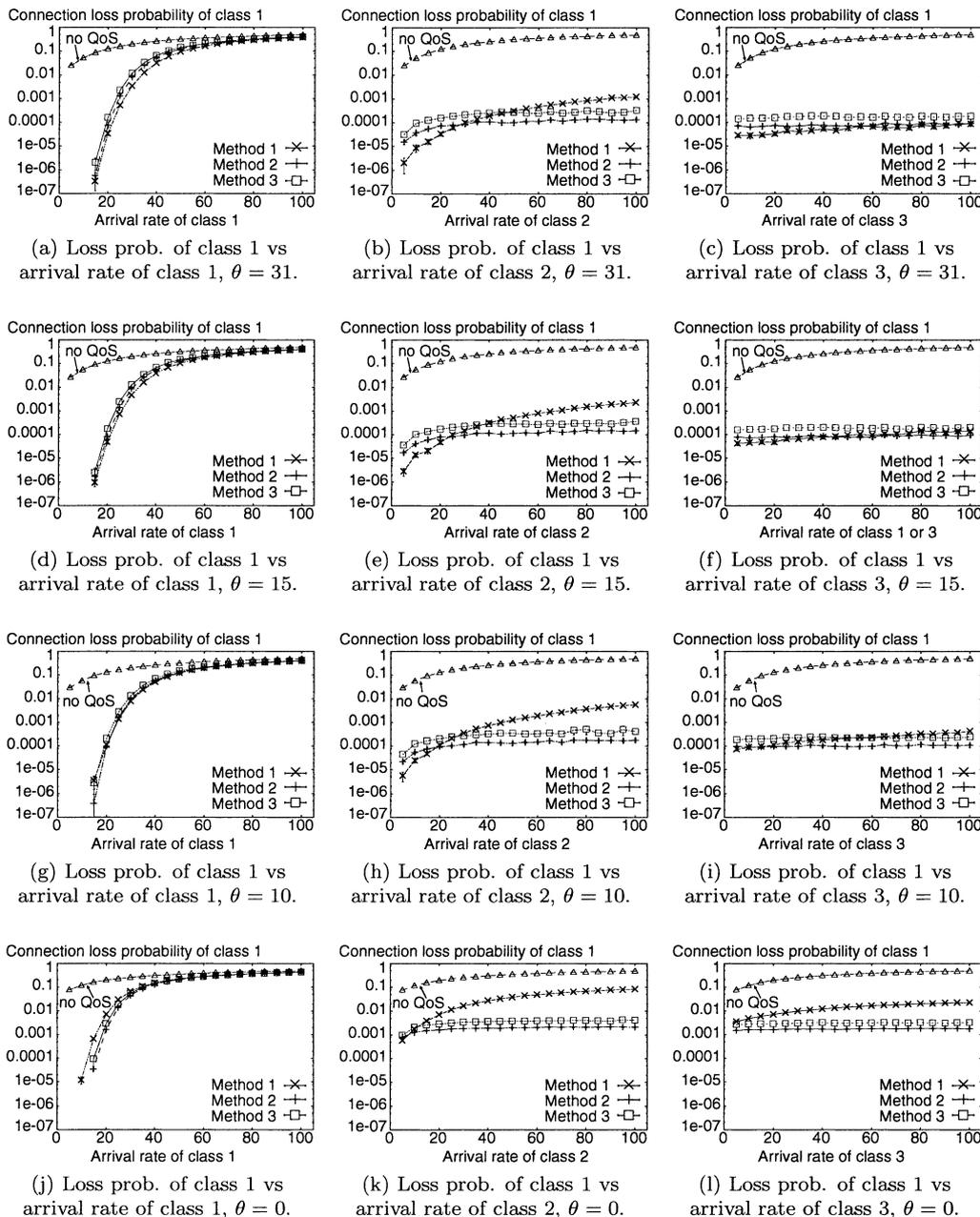


Fig. 7 Impact of threshold  $\theta$  for FWM wavelength conversion.

tion loss probabilities of low priority classes becomes large. When low priority class requires small connection loss probability, Method 1 is not effective. On the other hand, Method 2 is effective when several priority classes require small loss probabilities. Method 3 is effective when low priority class requires small loss probability.

The number of wavelengths and the wavelength conversion capability are also important factors for the connection loss probability. When both the number of wavelengths and the wavelength conversion capability are large, Method 1 can provide the smallest connection loss probability for class 1 while Method 2 can provide the smallest one for class 2. Moreover, Method 3 can provide the smallest connection

loss probability for class 3.

When the wavelength conversion capability is small, the robustness should be considered, too. Method 1 is affected by the arrival rates of low priority classes while Methods 2 and 3 are not affected so much. This robustness of Methods 2 and 3 are attractive for QoS provisioning in terms of connection loss probability.

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## Appendix: Equilibrium State Equations for Method 2

Let  $1_{\{X\}}$  denote the indicator function of event  $X$ , that is,

$$1_{\{X\}} = \begin{cases} 1, & \text{if } X \text{ occurs,} \\ 0, & \text{otherwise.} \end{cases}$$

When  $M = 3$  for Method 2, equilibrium state equations are as follows.

$$\lambda\pi(0, 0, 0) = \mu \{ \pi(1, 0, 0) + \pi(0, 1, 0) + \pi(0, 0, 1) \}, \quad (\text{A} \cdot 1)$$

$$\begin{aligned} (\lambda + N^{(1)}\mu)\pi(N^{(1)}, 0, 0) &= \mu \{ \pi(N^{(1)}, 1, 0) + \pi(N^{(1)}, 0, 1) \} \\ &+ 1_{\{N^{(1)} < \bar{w}^{(1)}\}}(N^{(1)} + 1)\mu\pi(N^{(1)} + 1, 0, 0) \\ &+ \lambda^{(1)}\pi(N^{(1)} - 1, 0, 0), \quad (N^{(1)} > 0), \quad (\text{A} \cdot 2) \end{aligned}$$

$$\begin{aligned} (\lambda + N^{(2)}\mu)\pi(0, N^{(2)}, 0) &= \mu \{ \pi(1, N^{(2)}, 0) + \pi(0, N^{(2)}, 1) \} \\ &+ 1_{\{N^{(2)} < \bar{w}^{(2)}\}}(N^{(2)} + 1)\mu\pi(0, N^{(2)} + 1, 0) \\ &+ \lambda^{(2)}\pi(0, N^{(2)} - 1, 0), \quad (N^{(2)} > 0), \quad (\text{A} \cdot 3) \end{aligned}$$

$$\begin{aligned} (\lambda + N^{(3)}\mu)\pi(0, 0, N^{(3)}) &= \mu \{ \pi(1, 0, N^{(3)}) + \pi(0, 1, N^{(3)}) \} \\ &+ 1_{\{N^{(3)} < \bar{w}^{(3)}\}}(N^{(3)} + 1)\mu\pi(0, 0, N^{(3)} + 1) \\ &+ (\lambda^{(2)} + \lambda^{(3)})\pi(0, 0, N^{(3)} - 1), \quad (N^{(3)} > 0), \quad (\text{A} \cdot 4) \end{aligned}$$

$$\begin{aligned} \{\lambda + (N^{(1)} + N^{(2)})\mu\}\pi(N^{(1)}, N^{(2)}, 0) &= \mu\pi(N^{(1)}, N^{(2)}, 1) \\ &+ 1_{\{N^{(1)} < \bar{w}^{(1)}\}}(N^{(1)} + 1)\mu\pi(N^{(1)} + 1, N^{(2)}, 0) \\ &+ 1_{\{N^{(2)} < \bar{w}^{(2)}\}}(N^{(2)} + 1)\mu\pi(N^{(1)}, N^{(2)} + 1, 0) \\ &+ \lambda^{(1)}\pi(N^{(1)} - 1, N^{(2)}, 0) \\ &+ 1_{\{N^{(1)} = \bar{w}^{(1)}\}}\lambda^{(1)}\pi(N^{(1)}, N^{(2)} - 1, 0), \quad (N^{(1)}, N^{(2)} > 0), \quad (\text{A} \cdot 5) \end{aligned}$$

$$\begin{aligned} \{\lambda + (N^{(2)} + N^{(3)})\mu\}\pi(0, N^{(2)}, N^{(3)}) &= \mu\pi(1, N^{(2)}, N^{(3)}) \\ &+ 1_{\{N^{(2)} < \bar{w}^{(2)}\}}(N^{(2)} + 1)\mu\pi(0, N^{(2)} + 1, N^{(3)}) \\ &+ 1_{\{N^{(3)} < \bar{w}^{(3)}\}}(N^{(3)} + 1)\mu\pi(0, N^{(2)}, N^{(3)} + 1) \\ &+ (\lambda^{(2)} + \lambda^{(3)})\pi(0, N^{(2)}, N^{(3)} - 1) \\ &+ 1_{\{N^{(3)} = \bar{w}^{(3)}\}}\lambda^{(2)}\pi(0, N^{(2)} - 1, N^{(3)}), \quad (N^{(2)}, N^{(3)} > 0), \quad (\text{A} \cdot 6) \end{aligned}$$

$$\begin{aligned} \{\lambda + (N^{(1)} + N^{(3)})\mu\}\pi(N^{(1)}, 0, N^{(3)}) &= \mu\pi(N^{(1)}, 1, N^{(3)}) \\ &+ 1_{\{N^{(1)} < \bar{w}^{(1)}\}}(N^{(1)} + 1)\mu\pi(N^{(1)} + 1, 0, N^{(3)}) \\ &+ 1_{\{N^{(3)} < \bar{w}^{(3)}\}}(N^{(3)} + 1)\mu\pi(N^{(1)}, 0, N^{(3)} + 1) \\ &+ \lambda^{(1)}\pi(N^{(1)} - 1, 0, N^{(3)}) \\ &+ (\lambda^{(2)} + \lambda^{(3)})\pi(N^{(1)}, 0, N^{(3)} - 1), \quad (N^{(1)}, N^{(3)} > 0), \quad (\text{A} \cdot 7) \end{aligned}$$

$$\begin{aligned} \{1_{\{\Gamma\}}\lambda^{(1)} + 1_{\{\Theta\}}\lambda^{(2)} + 1_{\{N^{(3)} < \bar{w}^{(3)}\}}\lambda^{(3)}\} \\ + (N^{(1)} + N^{(2)} + N^{(3)})\mu\}\pi(N^{(1)}, N^{(2)}, N^{(3)}) \\ = 1_{\{N^{(1)} < \bar{w}^{(1)}\}}(N^{(1)} + 1)\mu\pi(N^{(1)} + 1, N^{(2)}, N^{(3)}) \\ + 1_{\{N^{(2)} < \bar{w}^{(2)}\}}(N^{(2)} + 1)\mu\pi(N^{(1)}, N^{(2)} + 1, N^{(3)}) \\ + 1_{\{N^{(3)} < \bar{w}^{(3)}\}}(N^{(3)} + 1)\mu\pi(N^{(1)}, N^{(2)}, N^{(3)} + 1) \\ + \lambda^{(1)}\pi(N^{(1)} - 1, N^{(2)}, N^{(3)}) \\ + (\lambda^{(2)} + \lambda^{(3)})\pi(N^{(1)}, N^{(2)}, N^{(3)} - 1) \\ + 1_{\{N^{(1)} = \bar{w}^{(1)}\}}\lambda^{(1)}\pi(N^{(1)}, N^{(2)} - 1, N^{(3)}) \\ + 1_{\{N^{(3)} = \bar{w}^{(3)}\}}\lambda^{(2)}\pi(N^{(1)}, N^{(2)} - 1, N^{(3)}) \\ + 1_{\{N^{(1)} = \bar{w}^{(1)}, N^{(2)} = \bar{w}^{(2)}\}}\lambda^{(1)}\pi(N^{(1)}, N^{(2)}, N^{(3)} - 1), \quad (N^{(1)}, N^{(2)}, N^{(3)} > 0). \quad (\text{A} \cdot 8) \end{aligned}$$

In (A · 8), the sets of events  $\Gamma$  and  $\Theta$  are given by

$$\begin{aligned} \Gamma &= \{N^{(1)} < \bar{w}^{(1)}\} \cup \{N^{(2)} < \bar{w}^{(2)}\} \cup \{N^{(3)} < \bar{w}^{(3)}\}, \\ \Theta &= \{N^{(2)} < \bar{w}^{(2)}\} \cup \{N^{(3)} < \bar{w}^{(3)}\}. \end{aligned}$$



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